A Deterministic Inventory Model for Deteriorating Items with Linear Demand

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Abstract

The present paper deals with an inventory model for deteriorating items in which planning horizon is finite. The deterioration is taken as time dependent, demand is linear function of time and production rate depends both on inventory level & demand. The analytical development is provided to obtain the optimal solution to minimize the total cost per time unit of an inventory control system. Numerical analysis has been presented to accredit the validity of the mentioned model.

Keywords: Inventory model, Deteriorating items, linear demand

INTRODUCTION

Inventory modeling is a very important subject in logistics. Usually the analysis of inventory system is carried out without considering the effect of deterioration cannot be disregarded in inventory system. Deterioration means damage, spoilage, dryness, vaporization, etc. The products like fresh food i.e. Fruits, Vegetables, Fish, Meat, Photographic film, Batteries, Human blood, Photographic film, are knows as perishable products. Many researchers have studied the inventory problem with finite or infinite production rate by taking various production rates by taking various type of demand. T.K. Datta (1992) and Donaldson W.A. (1977) discussed on inventory problem for finite production rate with linear trend in demand. Balkhi and Benkheraur (1996) developed a production lot size inventory model with arbitrary production and demand rate depends on time function Bhunia and Maiti (1997) presented two deterministic inventory models in their paper the two types of production rates. In first model they considered that production rate depends on the on hand inventory. For second model they assumed that production rate depends on demand rate and demand is linearly change with time. Shortage and deterioration are not considered by them. Su.Ct. *et. al.* (1999)assumed that the production rate depends on demand and demand is exponentially decreasing function of time deterioration is constant and shortages are allowed in their model. Sharma and Sharma (2002) discussed an inventory model by taking the assumption that demand rate depends on inventory level and production rate depends on demand. Deterioration is constant and shortages are followed. kumar and Sharma presented a paper under more realistic situation for deteriorating items by assuming that the production rate is a linear combination of on hand inventory and demand. The demand is exponentially decreasing with time and deterioration is constant.

In the present study, we discussed a model in which production rate depends on both demand and on hand inventory. Deterioration is taken as time dependent and demand is consider as a linear function of time. The objective is to minimize the total cost per time of an inventory system.

ASSUMPTION AND NOTATIONS

Assumptions:

(i) Demand rate $D(t) = a + bt, a$ and b are constants, $a > 0, b \geq 0$.

(ii) Production rate $P(t) = K - \beta I(t) + \gamma D(t), K > 0, 0 \leq \beta < 1, 0 \leq \gamma < 1, P(t) > D(t)$

(iii) Deterioration θ is constant.

(iv) No replacement or repair of deteriorated item is made during a cycle.

(v) Shortages are allowed and fully backlogged.

Notations:

$I(t) =$ Inventory level at any time t, $t \geq 0$

$\theta =$ The items deterioration rate.

$I_m =$ Maximum inventory level.

$I_b =$ Unfilled order backlog.

$C_1 =$ Inventory carrying cost per unit per unit of time.

$C_d =$ The deterioration cost.

$C_3 =$ Setup cost for each new cycle.

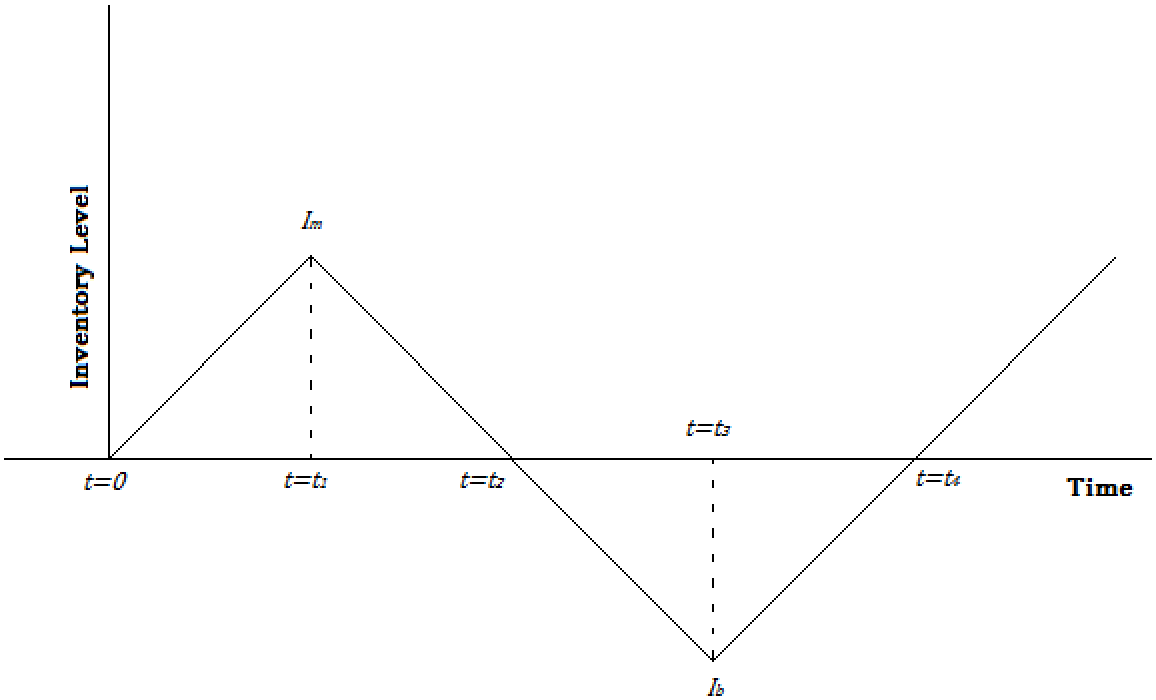
$C_s =$ Shortage cost per unit.

$T =$ Cycle time = $(t_1 + t_2 + t_3 + t_4)$

$C =$ The total average cost of the system.

MATHEMATICAL FORMULATION

Initially the stock level is zero. The production starts at a time $t = 0$, and after $t_1$units of time it reaches to maximum inventory level I_m. The production then stopped and the inventory level decreases due to demand and deterioration both, till it becomes again zero at $t = t_2$. At this time shortages start developing at $t = t_3$ it reaches to I_b, maximum shortage level. At this time fresh production start to clear the backlog by the time $t = t_4$. Our purpose is to find out the optimal value of t_1, t_2, t_3, t_4, I_m and I_b that minimize C over the time horizon $(0, T)$. The graphical representation of the model is shown below:



Let $I(t)$ be the inventory level at time t. The differential equations governing the stock during the period $(0, T)$ are.

$$\frac{dI(t)}{dt} + \theta I(t) = P(t) - D(t) \qquad 0 \le t \le t_1 \qquad (1.1.1)$$

$$\frac{dI(t)}{dt} + \theta I(t) = -D(t) \qquad 0 \le t \le t_2 \qquad (1.1.2)$$

$$\frac{dI(t)}{dt} = -D(t) \qquad 0 \le t \le t_3 \qquad (1.1.3)$$

$$\frac{dI(t)}{dt} = P(t) - D(t) \qquad 0 \le t \le t_4 \qquad (1.1.4)$$

And Initial conditions are

$I(t) = 0$ at $t = 0, t_1+t_2$, T (1.1.5)

$I(t) = I_m$ and $-I(t_1+t_2+t_2) = I_b$ (1.1.6)

On putting the value of P(t) and D(t), the equations (1.1.1) to (1.1.4) becomes

$$\frac{dI(t)}{dt} + \theta I(t) = K - \beta I(t) + (\gamma - 1)(a + bt) \qquad 0 \le t \le t_1 \qquad (1.1.7)$$

$$\frac{dI(t)}{dt} + \theta I(t) = -(a + bt) \qquad 0 \le t \le t_2 \qquad (1.1.8)$$

$$\frac{dI(t)}{dt} = -(a + bt) \qquad 0 \le t \le t_3 \qquad (1.1.9)$$

$$\frac{dI(t)}{dt} = K - \beta I(t) + (\gamma - 1)(a + bt) \qquad 0 \le t \le t_4 \qquad (1.1.10)$$

Solution of Equations:

Solution of equation (6.4.7) is given by

$$I(t)e^{(\theta+\beta)t} = \int \{K + (\gamma - 1)(a + bt)\}\, e^{(\theta+\beta)t} dt + K_1$$

(Where K_1 is the constant of integration)

$$\Rightarrow I(t)e^{(\theta+\beta)t} = \frac{Ke^{(\theta+\beta)t}}{(\theta+\beta)} + (\gamma - 1)\left\{\frac{ae^{(\theta+\beta)t}}{(\theta+\beta)} + \frac{bte^{(\theta+\beta)t}}{(\theta+\beta)} - \frac{be^{(\theta+\beta)t}}{(\theta+\beta)^2}\right\} + K_1$$

Now at $t = 0, I(t) = 0$, then

$$K_1 = -\left[\frac{K}{(\theta+\beta)} + (\gamma - 1)\left\{\frac{a}{(\theta+\beta)} - \frac{b}{(\theta+\beta)^2}\right\}\right]$$

$$\Rightarrow I(t)e^{(\theta+\beta)t} = \frac{Ke^{(\theta+\beta)t}}{(\theta+\beta)} + (\gamma - 1)\left\{\frac{a}{(\theta+\beta)} + \frac{bt}{(\theta+\beta)} - \frac{b}{(\theta+\beta)^2}\right\} + e^{(\theta+\beta)t}$$
$$-\left[\frac{K}{(\theta+\beta)} + (\gamma - 1)\left\{\frac{a}{(\theta+\beta)} - \frac{b}{(\theta+\beta)^2}\right\}\right]$$

or $I(t) = \dfrac{K}{(\theta+\beta)} + (\gamma - 1)\left\{\dfrac{a}{(\theta+\beta)} + \dfrac{bt}{(\theta+\beta)} - \dfrac{b}{(\theta+\beta)^2}\right\}$

$$-\left[\frac{K}{(\theta+\beta)} + (\gamma - 1)\left\{\frac{a}{(\theta+\beta)} - \frac{b}{(\theta+\beta)^2}\right\}\right] e^{(\theta+\beta)t} \qquad 0 \le t \le t_1 \qquad (1.2.1)$$

Solution of equation (1.1.8) is given by

$$I(t)e^{\theta t} = -\int (a + bt)e^{\theta t} dt + K_2$$

(where K_2 is the constant of integration)

$$\Rightarrow I(t)e^{\theta t} = \left\{\frac{ae^{\theta t}}{\theta} + \frac{bte^{\theta t}}{\theta} - \frac{be^{\theta t}}{\theta^2}\right\} + K_2$$

Now at $t = t_2, I(t) = 0$ then

$$K_2 = \left\{\frac{ae^{\theta t_2}}{\theta} + \frac{bt_2 e^{\theta t_2}}{\theta} - \frac{be^{\theta t_2}}{\theta^2}\right\}$$

Therefore

$$I(t)e^{\theta t} = \left\{\frac{ae^{\theta t}}{\theta} + \frac{bte^{\theta t}}{\theta} - \frac{be^{\theta t}}{\theta^2}\right\} + \left\{\frac{ae^{\theta t_2}}{\theta} + \frac{bt_2 e^{\theta t_2}}{\theta} - \frac{be^{\theta t_2}}{\theta^2}\right\}$$

or $I(t) = -\left\{\dfrac{a}{\theta} + \dfrac{bt}{\theta} - \dfrac{b}{\theta^2}\right\} + \left\{\dfrac{a}{\theta} + \dfrac{bt_2}{\theta} - \dfrac{b}{\theta^2}\right\} e^{\theta(t_2 - t)} \qquad 0 \le t \le t_2 \qquad (1.2.2)$

Solution of equation (1.1.9) is given by

$$I(t) = -\int (a + bt)dt + K_3$$

(where K_3 is constant of integration)

$$\Rightarrow I(t) = -\left\{at + \frac{bt^2}{2}\right\} + K_3$$

at $t = 0, I(t) = 0\ then\ K_3 = 0$

Therefore

$$I(t) = -\left\{at + \frac{bt^2}{2}\right\} \qquad 0 \le t \le t_3 \qquad (1.2.3)$$

Solution of equation (1.1.10) is given by

$$I(t)e^{\beta t} = \int \{K + (\gamma - 1)(a + bt)\}e^{\beta t}dt + K_4$$

(where K_4 is constant of integration)

$$\Rightarrow I(t)e^{\beta t} = \frac{Ke^{\beta t}}{\beta} + (\gamma - 1)\left\{\frac{ae^{\beta t}}{\beta} + \frac{bte^{\beta t}}{\beta} - \frac{be^{\beta t}}{\beta^2}\right\} + K_4$$

Now at $t = t_4,\ I(t) = 0$ then

$$K_4 = -\left[\frac{Ke^{\beta t_4}}{\beta} + (\gamma - 1)\left\{\frac{ae^{\beta t_4}}{\beta} + \frac{bt_4e^{\beta t_4}}{\beta} - \frac{be^{\beta t_4}}{\beta^2}\right\}\right]$$

Therefore

$$I(t) = \frac{K}{\beta} + (\gamma - 1)\left\{\frac{a}{\beta} + \frac{bt}{\beta} - \frac{b}{\beta^2}\right\} - \left[\frac{K}{\beta} + (\gamma - 1)\left\{\frac{a}{\beta} + \frac{bt_4}{\beta} - \frac{b}{\beta^2}\right\}\right]e^{(t_4 - t)} \quad 0 \le t \le t_4 \quad (1.2.4)$$

Now using the initial condition $I(t_1) = I_m$ in equation (1.2.1) and (1.2.2) we get

$$I_m = \frac{K}{(\theta + \beta)} + (\gamma - 1)\left\{\frac{a}{(\theta + \beta)} + \frac{bt_1}{(\theta + \beta)} - \frac{b}{(\theta + \beta)^2}\right\} - \left[\frac{K}{(\theta + \beta)} + (\gamma - 1)\left\{\frac{a}{(\theta + \beta)} - \frac{b}{(\theta + \beta)^2}\right\}\right]e^{(\theta+\beta)t_1}$$

$$= -\left\{\frac{a}{\theta} - \frac{b}{\theta^2}\right\} + \left\{\frac{a}{\theta} + \frac{bt_2}{\theta} - \frac{b}{\theta^2}\right\}e^{\theta t_2} \qquad (1.2.5)$$

or $$\frac{K}{(\theta + \beta)} + (\gamma - 1)\left\{\frac{a}{(\theta + \beta)} + \frac{bt_1}{(\theta + \beta)} - \frac{b}{(\theta + \beta)^2}\right\} - \left[\frac{K}{(\theta + \beta)} + (\gamma - 1)\left\{\frac{a}{(\theta + \beta)} - \frac{b}{(\theta + \beta)^2}\right\}\right]e^{(\theta+\beta)t_1}$$

$$= -\left\{\frac{a}{\theta} - \frac{b}{\theta^2}\right\} + \left\{\frac{a}{\theta} + \frac{bt_2}{\theta} - \frac{b}{\theta^2}\right\}\left\{1 + \theta t_2 + \frac{\theta^2 {t_2}^2}{2}\right\}$$

[neglecting higher terms of θ]

or $$\frac{K}{(\theta + \beta)} + (\gamma - 1)\left\{\frac{a}{(\theta + \beta)} + \frac{bt_1}{(\theta + \beta)} - \frac{b}{(\theta + \beta)^2}\right\} - \left[\frac{K}{(\theta + \beta)} + (\gamma - 1)\left\{\frac{a}{(\theta + \beta)} - \frac{b}{(\theta + \beta)^2}\right\}\right]e^{(\theta+\beta)t_1} = at_2 + \frac{(b + a\theta){t_2}^2}{2}$$

$$\Rightarrow t_2 = -\frac{\sqrt[a+]{a^2 + 2(b + a\theta)\left[\frac{K}{(\theta + \beta)} + (\gamma - 1)\left\{\frac{a}{(\theta + \beta)} + \frac{bt_1}{(\theta + \beta)} - \frac{b}{(\theta + \beta)^2}\right\} - \left[\frac{K}{(\theta + \beta)} + (\gamma - 1)\left\{\frac{a}{(\theta + \beta)} - \frac{b}{(\theta + \beta)^2}\right\}\right]e^{(\theta+\beta)t_1}\right]}}{(b + a\theta)} \qquad (1.2.6)$$

or $t_2 = R(t_1)$

Now on using the initial condition $-I(t_1 + t_2 + t_3) = I_b$ in equation (6.5.3) and (6.5.4) we get

$$I_b = at_3 + \frac{b{t_3}^2}{2} = \frac{K}{\beta} + (\gamma - 1)\left\{\frac{a}{\beta} - \frac{b}{\beta^2}\right\} - \left[\frac{K}{\beta} + (\gamma - 1)\left\{\frac{a}{\beta} + \frac{bt_4}{\beta} - \frac{b}{\beta^2}\right\}\right]e^{\beta t_4} \qquad (1.2.7)$$

$$\Rightarrow t_3 = -\frac{a+\sqrt{a^2 + 2b\left[\frac{K}{\beta} + (\gamma-1)\left\{\frac{a}{\beta} - \frac{b}{\beta^2}\right\} - \left\{\frac{K}{\beta} + (\gamma-1)\left\{\frac{a}{\beta} + \frac{bt_4}{\beta} - \frac{b}{\beta^2}\right\}\right\} e^{\beta t_4}\right]}}{b} \tag{1.2.8}$$

$$\Rightarrow t_3 = R(t_4)$$

Let the inventory during this period is H then

$$H = \int_0^{t_1} I(t)dt + \int_0^{t_2} I(t)dt$$

From equation (1.2.1)

$$\int_0^{t_1} I(t)dt = \int_0^{t_1} \left[\frac{K}{(\theta+\beta)} + (\gamma-1)\left\{\frac{a}{(\theta+\beta)} + \frac{bt}{(\theta+\beta)} - \frac{b}{(\theta+\beta)^2}\right\}\right] dt$$
$$+ \int_0^{t_1} \left[\frac{K}{(\theta+\beta)} + (\gamma-1)\left\{\frac{a}{(\theta+\beta)} - \frac{b}{(\theta+\beta)^2}\right\}\right] e^{-(\theta+\beta)t} dt$$

$$\Rightarrow \int_0^{t_1} I(t)dt = \left[\frac{Kt}{(\theta+\beta)} + (\gamma-1)\left\{\frac{at}{(\theta+\beta)} + \frac{bt^2}{2(\theta+\beta)} - \frac{bt}{(\theta+\beta)^2}\right\}\right]_0^{t_1}$$
$$- \left[\frac{Kt}{(\theta+\beta)} + (\gamma-1)\left\{\frac{a}{(\theta+\beta)} - \frac{b}{(\theta+\beta)^2}\right\} \frac{e^{-(\theta+\beta)t}}{\{-(\theta+\beta)\}}\right]_0^{t_1}$$

$$\Rightarrow \int_0^{t_1} I(t)dt = \frac{K}{(\theta+\beta)^2}\left\{(\theta+\beta)t_1 + e^{-(\theta+\beta)t_1} - 1\right\} + \frac{(\gamma-1)a}{(\theta+\beta)^2}\left\{(\theta+\beta)t_1 + e^{-(\theta+\beta)t_1} - 1\right\}$$
$$- \frac{(\gamma-1)b}{(\theta+\beta)^3}\left\{-\frac{(\theta+\beta)^2 {t_1}^2}{2} + (\theta+\beta)t_1 + e^{-(\theta+\beta)t_1} - 1\right\} \tag{A_1}$$

From equation (6.5.2)

$$\int_0^{t_2} I(t)dt = \int_0^{t_2} \left[-\left\{\frac{a}{\theta} + \frac{bt}{\theta} - \frac{b}{\theta^2}\right\} + \left\{\frac{a}{\theta} + \frac{bt_2}{\theta} - \frac{b}{\theta^2}\right\} e^{\theta(t_2-t)}\right] dt$$

$$= -\left\{\frac{at}{\theta} + \frac{bt^2}{\theta} - \frac{bt}{\theta^2}\right\}_0^{t_1} + \left[\left\{\frac{a}{\theta} + \frac{bt_2}{\theta} - \frac{b}{\theta^2}\right\} \frac{e^{\theta(t_2-t)}}{(-\theta)}\right]_0^{t_2}$$

$$\Rightarrow \int_0^{t_2} I(t)dt = \frac{a}{\theta^2}\left\{-\theta t_2 - 1 + e^{\theta t_2}\right\} + \frac{b}{\theta^3}\left\{\frac{-\theta^2 {t_2}^2}{2} + \theta t_2 e^{\theta t_2} + 1 - e^{\theta t_2}\right\} \tag{A_2}$$

From (A_1) and (A_2) we get

$$H = \frac{K}{(\theta+\beta)^2}\left\{(\theta+\beta)t_1 + e^{-(\theta+\beta)t_1} - 1\right\} + \frac{(\gamma-1)a}{(\theta+\beta)^2}\left\{(\theta+\beta)t_1 + e^{-(\theta+\beta)t_1} - 1\right\}$$
$$- \frac{b(\gamma-1)}{(\theta+\beta)^3}\left\{-\frac{(\theta+\beta)^2 {t_1}^2}{2} + (\theta+\beta)t_1 + e^{-(\theta+\beta)t_1} - 1\right\}$$
$$+ \frac{a}{\theta^2}\left\{-\theta t_2 - 1 + e^{\theta t_2}\right\} + \frac{b}{\theta^3}\left\{\frac{-\theta^2 {t_2}^2}{2} + \theta t_2 e^{\theta t_2} + 1 - e^{\theta t_2}\right\} \tag{A}$$

The deteriorated units during this period = θH

Therefore deterioration cost for the period $(0, T)$ is = $C_d \theta H$ (1.2.9)

and inventory holding cost over $(0, T)$ is = $C_1 H$ (1.2.10)

Where H is given by equation (A)

The shortage cost is

$$= C_S\left[-\int_0^{t_3} I(t)dt + \int_0^{t_4} I(t)dt\right]$$

On putting the value from equation (1.2.3) and (1.2.4) it is

$$= C_S\left[-\int_0^{t_3}\left\{at + \frac{bt^2}{2}\right\}dt + \int_0^{t_4}\left[\frac{K}{\beta} + (\gamma-1)\left\{\frac{a}{\beta} + \frac{bt}{\beta} - \frac{b}{\beta^2}\right\}\right]dt\right]$$

$$- \int_0^{t_4}\left[\frac{K}{\beta} + (\gamma-1)\left\{\frac{a}{\beta} + \frac{bt_4}{\beta} - \frac{b}{\beta^2}\right\}\right]e^{(t_4-t)}dt$$

$$= C_S\left[\left\{\frac{at^2}{2} + \frac{bt^3}{6}\right\}_0^{t_3} + \left\{\frac{Kt}{\beta} + (\gamma-1)\left(\frac{at}{\beta} + \frac{bt^2}{2\beta} - \frac{bt}{\beta^2}\right)\right\}_0^{t_4}\right.$$

$$\left. - \left\{\left[\frac{K}{\beta} + (\gamma-1)\left(\frac{a}{\beta} + \frac{bt_4}{\beta} - \frac{b}{\beta^2}\right)\right]\frac{e^{\beta(t_4-t)}}{(-\beta)}\right\}_0^{t_4}\right]$$

$$\Rightarrow C_S\left[-\int_0^{t_3} I(t)dt + \int_0^{t_4} I(t)dt\right]$$

$$= C_S\left[\frac{a{t_3}^2}{2} + \frac{b{t_3}^3}{6} + \frac{K}{\beta^2}\{\beta t_4 + 1 - e^{\beta t_4}\} + \frac{(\gamma-1)a}{\beta^2}\{\beta t_4 + 1 - e^{\beta t_4}\}\right.$$

$$\left. + \frac{(\gamma-1)b}{\beta^3}\left\{\frac{\beta^2{t_4}^2}{2} - t_4\beta e^{\beta t_4} - 1 - e^{\beta t_4}\right\}\right] \qquad (1.2.11)$$

Hence total average cost of the system is

C = Setup cost + Deterioration Cost + Inventory Carrying Cost + Shortage Cost

On putting the values from equations (A), (1.2.9) (1.2.10) and (1.2.11) we get

$$C = \frac{C_3}{T} + \frac{(\theta C_d + C_1)}{T}\left[\frac{K}{(\theta+\beta)^2}\{(\theta+\beta)t_1 + e^{-(\theta+\beta)t_1} - 1\}\right.$$

$$+ \frac{(\gamma-1)a}{(\theta+\beta)^2}\{(\theta+\beta)t_1 + e^{-(\theta+\beta)t_1} - 1\}$$

$$+ \frac{(\gamma-1)b}{(\theta+\beta)^3}\left\{-\frac{(\theta+\beta)^2{t_1}^2}{2} + (\theta+\beta)t_1 + e^{-(\theta+\beta)t_1} - 1\right\}$$

$$\left. + \frac{a}{\theta^2}\{-\theta t_2 - 1 + e^{\theta t_2}\} + \frac{b}{\theta^3}\left\{\frac{-\theta^2{t_2}^2}{2} + \theta t_2 e^{\theta t_2} + 1 - e^{\theta t_2}\right\}\right]$$

$$+ \frac{C_S}{T}\left[\frac{a{t_3}^2}{2} + \frac{b{t_3}^3}{6} + \frac{K}{\beta^2}\{\beta t_4 + 1 - e^{\beta t_4}\} + \frac{(\gamma-1)a}{\beta^2}\{\beta t_4 + 1 - e^{\beta t_4}\}\right.$$

$$\left. + \frac{(\gamma-1)b}{\beta^3}\left\{\frac{\beta^2{t_4}^2}{2} - t_4\beta e^{\beta t_4} - 1 - e^{\beta t_4}\right\}\right] \qquad (1.2.12)$$

Approximate Solution Procedure: According to equation (1.2.12) finding the optimal solution for this model is extremely difficult. Therefore it is reasonable to use maclaurin series for approximation. The problem as a whole can be simplified to the following equation.

$$e^{-xt} = 1 - xt + \frac{x^2t^2}{2} \tag{1.3.1}$$

By using equation (1.3.1) the total average cost of the system in this case, is simply

$$C = \frac{C_3}{T} + \frac{(\theta C_d + C_1)}{T}\left[\frac{Kt_1{}^2}{2} + \frac{(\gamma-1)at_1{}^2}{2} + \frac{bt_1{}^3}{6} + \frac{at_2{}^2}{2} + \frac{bt_2{}^3}{3}\right] + \frac{C_S}{T}\left[\frac{at_3{}^2}{2} + \frac{bt_3{}^3}{6} - \frac{Kt_4{}^2}{2} - \frac{(\gamma-1)at_4{}^2}{2} - \frac{(\gamma-1)bt_4{}^3}{3}\right] \tag{1.3.2}$$

Equation (1.3.2) contains four variables t_1, t_2, t_3 and t_4. However these values are not independent and are related by equation (1.2.6) and (1.2.8). Again $C > 0$, hence the optimal value of t_1 and t_4 which minimize total average cost are the solution of the equations

$$\frac{\partial C}{\partial t_1} = 0 \text{ and } \frac{\partial C}{\partial t_4} = 0 \tag{1.3.3}$$

Provided that these values of t_1 and t_4 satisfy the conditions.

$$\frac{\partial^2 C}{\partial t_1{}^2} > 0, \quad \frac{\partial^2 C}{\partial t_4{}^2} > 0 \quad and \quad \frac{\partial^2 C}{\partial t_1{}^2} \times \frac{\partial^2 C}{\partial t_4{}^2} - \frac{\partial^2 C}{\partial t_1 \partial t_4} > 0$$

Now differentiating equation (1.3.2) with respect to t_1 and t_4 and equating them to zero we get,

$$\frac{-C_3}{T^2}[1 + R'(t_1)] - \frac{(\theta C_d + C_1)}{T^2}[1 + R'(t_1)] \left[\frac{Kt_1{}^2}{2} + \frac{(\gamma-1)at_1{}^2}{2} + \frac{bt_1{}^3}{6} + \frac{aR^2(t_1)}{2} + \frac{bR^3(t_1)}{3}\right] + \frac{(\theta C_d + C_1)}{T}\left[Kt_1 + (\gamma-1)at_1 + \frac{bt_1{}^2}{2} + aR(t_1)R'(t_1) + bR^2(t_1)R'(t_1)\right] - \frac{C_S[1 + R'(t_1)]}{T^2}\left[\frac{aR^2(t_4)}{2} + \frac{bR^3(t_4)}{6} - \frac{(\gamma-1)at_4{}^2}{2} - \frac{Kt_4{}^2}{2} - \frac{(\gamma-1)bt_4{}^3}{3}\right] \tag{1.3.4}$$

and

$$\frac{-C_3}{T^2}[1 + R'(t_4)] - \frac{(\theta C_d + C_1)}{T^2}[1 + R'(t_4)] \left[\frac{Kt_1{}^2}{2} + \frac{(\gamma-1)at_1{}^2}{2} + \frac{bt_1{}^3}{6} + \frac{aR^2(t_1)}{2} + \frac{bR^3(t_1)}{3}\right] - \frac{C_S[1 + R'(t_4)]}{T^2}\left[\frac{aR^2(t_4)}{2} + \frac{bR^3(t_4)}{6} - \frac{Kt_4{}^2}{2} - \frac{(\gamma-1)at_4{}^2}{2} - \frac{(\gamma-1)bt_4{}^3}{3}\right] + \frac{C_S}{T}\left[aR(t_4)R'(t_4) + \frac{bR^2(t_4)R'(t_4)}{2} - Kt_4 - (\gamma-1)at_4 - (\gamma-1)bt_4{}^2\right] = 0 \tag{1.3.5}$$

The optimum value of t_2, t_3, I_m, I_b and the minimum total average cost C can be calculated from equation (1.2.6), (1.2.8), (1.2.5), (1.2.7) and (1.5.12) respectively.

Special Case: If we put $K = \alpha, \gamma = 0, C_d = 0$ and $C_s = 0$ then equation (1.2.12) reduces into the following from:

$$C = \frac{C_3}{T} + \frac{C_1}{T}\left[\frac{\alpha}{(\theta+\beta)^2}\left\{(\theta+\beta)t_1 + e^{-(\theta+\beta)t_1} - 1\right\} + \frac{a}{(\theta+\beta)^2}\left\{(\theta+\beta)t_1 + e^{-(\theta+\beta)t_1} - 1\right\} + \frac{b}{(\theta+\beta)^3}\left\{-\frac{(\theta+\beta)^2{t_1}^2}{2} + (\theta+\beta)t_1 + e^{-(\theta+\beta)t_1} - 1\right\} + \frac{a}{\theta^2}\left\{-\theta t_2 - 1 + e^{\theta t_2}\right\} + \frac{b}{\theta^3}\left\{\frac{-\theta^2{t_2}^2}{2} + \theta t_2 e^{\theta t_2} + 1 - e^{\theta t_2}\right\}\right]$$

Now put $\theta = 0$ then

$$C = \frac{C_3}{T} + \frac{C_1}{T}\left[\frac{(\alpha - a)t_1}{\beta} - \frac{b{t_1}^2}{2\beta} + \frac{bt_1}{\beta^2} + \frac{e^{\beta t_1}}{\beta} - \frac{\mu}{\beta} + \frac{a{t_2}^2}{2} + \frac{b{t_2}^3}{2}\right]$$

Where $\mu = \frac{\alpha - a}{\beta} + \frac{b}{\beta^2}$

Therefore on putting $K = \alpha, \gamma = 0, C_d = 0, C_S = 0$ and $\theta \to 0$ this model reduces into Bhunia & Maiti '& [10] model.

This study presents a production inventory model for deterioration items in which production rate depends on both on hand inventory and demand. The demand is linearly increasing function of time. The present model is developed under more realistic assumptions and provides valuable reference for decision makers in planning the production and controlling the inventory.

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